Study on Option Pricing Based on Black-Scholes Model

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Abstract: With the rapid development of global financial markets, options are getting more and more attention from many people. It is necessary to conduct more in-depth research on options.Based on the Black-Scholes model, this paper studies the pricing and calculation of European options, and obtains the Black-Scholes formula by solving the Black-Scholes equation.The numerical calculation method is used to solve the European option pricing. The derivation process of the finite difference method is analyzed, including the inner limit difference method, the extrapolation finite difference method and the Crank-Nicolson difference method.Finally, the final option value is obtained to introduce the initial option value, and an instance analysis is performed to realize the combination of mathematical knowledge and computer language.

1. Introduction

At present, how to effectively control the option risk has been related to whether the option development can be transferred from the research stage to the trial operation stage.However, in order to effectively manage and control the option risk, the option must be reasonably priced first.Therefore, the study of option pricing methods is more important.

In 1900, French financial expert Louis Bachelier published the first thesis "Theorie de la Speculation" on option pricing [1], which was recognized as a milestone in modern finance, and he was the first in his paper. A stochastic model of stock price operation is proposed by using random walk idea.In 1964, Paul Samuelson revised the model of Louis Bachelier to replace the stock price in the original model with the return of stocks.In 1973, Fischer Black and Myron Scholes published the paper "The pricing of options and corporate liabilities" [2], in which they established a call option pricing formula.In 1979, Cox, J., S. Ross and M. Rubinstein published the paper "Option Pricing: A Simplified Approach" in the Journal of Financial Economics [3], which proposed a simple option for discrete time. The pricing method is called the Cox-Ross-Rubinstein binomial option pricing model.

1 Behavioral Pattern of Stock Prices

Definition 1: The Markov process is a stochastic process that states that only the current value of a variable is related to future predictions.

It is often assumed that stock prices follow the Markov process, so the stock price behavior model usually uses a special form of the Markov random process, the Wiener process, also known as Brownian motion.We need to understand the variables that follow the Wiener process. z Behavior that can be considered at small intervals z the change in value.

Set a small interval length to Δt, definition Δz for Δt in time z the change.To make z follow the Wiener process, Δz must meet:

(1) Δz Versus Δt is the relationship satisfies the equation:

$$\Delta z = \varepsilon\sqrt{\Delta t}$$

Among them ε is a random value extracted from the N (0, l) distribution.

(2) For any two different time intervals Δt, Δz is the values are independent of each other.

Variable x generalized Wiener process dz definition is as follows:

$$dx = adt + bdz$$

Among them a , b is a constant.The expected value of the drift rate for the general Wiener process given in the above equation a Expected value of variance b^2 .

Theorem 1: Hypothetical variables x is the value follows the Ito process:

$$dx = \mu(x,t)dt + \sigma(x,t)dz$$

Among them dz is a Wiener process, μ with σ x and t function.variable x drift rate μ and variance rate σ^2 .Ito theorem shows x with t function G follow the process below:

$$dV_t = (\frac{\partial V}{\partial t}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2)dt + \frac{\partial G}{\partial x}bdz$$

Due to dz be the Wiener process, so G also follow the Ito process.

2. Black-Scholes Model

2.1 Finite difference method

To solve the differential equations satisfied by the derivative securities, the finite difference method can be used. The method is to transform the differential equation into a series of difference equations, and then use the iterative method to solve the difference equations.To illustrate this approach, we consider using it to estimate a European put option that does not pay dividends.

Proceed as follows:

Step 1: First determine the differential equation that the option satisfies:

$$\frac{\partial P}{\partial t} + rS\frac{\partial P}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 P}{\partial S^2} - rP = 0$$

Step 2: Assume that the duration of the option is T , divide this period into N equal interval, length is $\Delta t = T / N$ time interval.consider $N+1$ time points:

$$0, \Delta t, 2\Delta t, \cdots, T$$

Step 3: Assumption $S_{\max}$ is the highest price for the stock.Definition $\Delta S = S_{\max} / M$ and consider both $M+1$ stock prices:

$$0, \Delta S, 2\Delta S, \cdots, S$$

Therefore, the selected stock price and time constitute a common $(M+1)(N+1)$ grid of points.Points on the grid (i, j) corresponding to time $i\Delta t$ the stock price is $j\Delta S$ .Variable $P_{i,j}$ representative (i, j) the option price of the point.

Step 4: Calculate the above partial differential equation by using the inclusive difference method in the finite difference method, for the point inside the grid (i, j) , $\partial P / \partial S$ can be approximated as:

$$\frac{\partial P}{\partial S} = \frac{P_{i,j+1} - P_{i,j}}{\Delta S}$$

It is called forward difference approximation; or backward difference approximation. By averaging the above two difference equations, we can derive a symmetric difference equation:

$$\frac{\partial P}{\partial S} = \frac{P_{i,j+1} - P_{i,j-1}}{2\Delta S}$$

For $\partial P / \partial t$ using a forward differential approximation $i\Delta t$ Time price and $(i+1)\Delta t$ the price is related:

$$\frac{\partial P}{\partial t} = \frac{P_{i+1,j} - P_{i,j}}{\Delta t}$$

At the point $(i, j+1)$ of $\partial P / \partial S$ Backward difference approximation:

$$\frac{P_{i,j+1} - P_{i,j}}{\Delta S}$$

At the point (i, j), Correct $\partial^2 P / \partial S^2$ The finite difference approximation is:

$$\frac{\partial^2 P}{\partial S^2} = \partial(\frac{\partial P}{\partial S}) / \partial S = \frac{1}{\Delta S}(\frac{P_{i,j+1} - P_{i,j}}{\Delta S} - \frac{P_{i,j} - P_{i,j-1}}{\Delta S}) = \frac{P_{i,j+1} + P_{i,j-1} - 2P_{i,j}}{\Delta S^2}$$

Step 5: Combine the above multiple formulas, and $S = j\Delta S$, got:

$$\frac{P_{i+1,j} - P_{i,j}}{\Delta t} + rj\Delta S \frac{P_{i,j+1} - P_{i,j-1}}{2\Delta S} + \frac{1}{2}\sigma^2 j^2 \Delta S^2 \frac{P_{i,j+1} + P_{i,j-1} - 2P_{i,j}}{\Delta S^2} = rP_{i,j}$$

Among them:

$$j = 1, 2, \cdots, \ M-1, \ i = 0, 1\cdots, \ N-1$$

After the merger:

$$a_j P_{i,j-1} + b_j P_{i,j} + c_j P_{i,j+1} = P_{i+1,j}$$

$$a_j = \frac{1}{2} rj\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t$$

$$b_j = 1 + \sigma^2 j^2 \Delta t + r\Delta t$$

$$c_j = -\frac{1}{2} rj\Delta t - \frac{1}{2}\sigma^2 j^2 \Delta t$$

T Is the value of a put option at the moment is $\max[X - S_T, 0]$, among them S_T for T is the stock price of the moment, therefore:

$$P_{N,j} = \max[X - j\Delta S, 0]$$

$$j = 0, 1, 2, \cdots, \ M$$

When the stock price is zero, the value of the put is X , therefore:

$$P_{i,0} = X$$

$$i = 0, 1\cdots, \ N$$

When the stock price tends to infinity, the value of the put option tends to zero.So use the approximation:

$$P_{i,M} = j\Delta S$$

$$i = 0, 1\cdots, \ N$$

The above formula defines three boundaries (ie $S = 0, S = S_{\max}$ with $t = T$) the value of the put option, also requires the left border $P_{0,j}$ Value, one of the grid points is the option value we requested.Using boundary conditions, you can write $(N-1)\Delta t$ Moment of time $M-1$ Simultaneous equations:

$$a_j P_{N-1,j-1} + b_j P_{N-1,j} + c_j P_{N-1,j+1} = P_{N,j}$$

$$j = 1, 2, \cdots, \quad M-1$$

So solve each $P_{N-1,j}$ Value, and so on, can be calculated $P_{0,j}$, when $j\Delta S$ Corresponding to the initial asset price, the grid corresponds to P is the required option value.

2.2 Extrapolation of finite difference

The internal limit difference method is slightly modified, and the extrapolated finite difference method is used to assume the point. (i, j) At $\frac{\partial P}{\partial S}$ with $\frac{\partial^2 P}{\partial S^2}$ versus $(i, j+1)$ the corresponding values are equal, namely:

$$\frac{\partial P}{\partial S} = \frac{P_{i+1,j+1} - P_{i+1,j-1}}{2\Delta S}$$

$$\frac{\partial^2 P}{\partial S^2} = \frac{P_{i+1,j+1} + P_{i+1,j-1} - 2P_{i+1,j}}{\Delta S^2}$$

The corresponding difference equation is modified to:

$$a_j^* P_{i+1,j-1} + b_j^* P_{i+1,j} + c_j^* P_{i+1,j+1} = P_{i,j}$$

Among them:

$$a_j^* = \frac{1}{1+r\Delta t}\left(-\frac{1}{2} rj\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

$$b_j^* = \frac{1}{1+r\Delta t}(1 - \sigma^2 j^2 \Delta t)$$

$$c_j^* = \frac{1}{1+r\Delta t}\left(\frac{1}{2} rj\Delta t + \frac{1}{2}\sigma^2 j^2 \Delta t\right)$$

This is the dominant finite difference equation.

The advantage of the inclusive and extrapolated finite difference method in option pricing is mainly: when the lattice points are regular and uniform, it is relatively simple to convert a partial differential equation into a difference equation.Both the inclusive and extrapolated methods have their own advantages and disadvantages.The extrapolation method is relatively straightforward to calculate, and it is not necessary to solve a large number of simultaneous equations like the in-line method, and the workload is small and easy to apply.However, the extrapolation method has a defect: its three probabilities may be less than zero, which leads to the instability of this method, and its solution may not converge to the solution of the partial differential equation.The Crank-Nicolson difference format provided below is the average of the two methods.The included finite difference equation gives:

$$a_j P_{i-1,j-1} + b_j P_{i-1,j} + c_j P_{i-1,j+1} = P_{i,j}$$

The extrapolated finite difference equation gives:

$$a_j^* P_{i,j-1} + b_j^* P_{i,j} + c_j^* P_{i,j+1} = P_{i-1,j}$$

Average the two equations to get:

$$P_{i,j} + P_{i-1,j} = a_j P_{i-1,j-1} + b_j P_{i-1,j} + c_j P_{i-1,j+1} + a^*_j P_{i,j-1} + b^*_j P_{i,j} + c^*_j P_{i,j+1}$$

Make:

$$g_{i,j} = P_{i,j} - a^*_j P_{i,j-1} - b^*_j P_{i,j} - c^*_j P_{i,j+1},$$

Get:

$$g_{i,j} = a_j P_{i-1,j-1} + b_j P_{i-1,j} + c_j P_{i-1,j+1} - P_{i-1,j}$$

This shows that using Crank-Nicolson is similar to using the inner limit difference method.The advantage of the Crank-Nicolson method is that it converges faster than the intrinsic and extrapolated finite difference methods.

3. Analysis of Option Calculation Examples

This article considers a 5-month European put option that does not pay dividends. The stock price is 50 yuan, the execution price is 50 yuan, the risk-free rate is 10% per year, the volatility is 40% per year, and the present value of the option is sought.

If you use the Black-Scholes formula to calculate a value, but there is obviously a better way to solve it, use the computer language to describe the embedded difference method, and then cancel the value of this European put option through the computer.make M 、 N with ΔS The values are taken as 20, 10 and 5 respectively, known:

$$S = 50, X = 50,$$

$$\sigma = 0.4 . r = 0.1$$

According to the above values, the current value of the European put option is 3.9113 yuan, and the option price calculated by the Black-Scholes formula is 4.08 yuan, but the gap between the two is still a bit large, so it is necessary to continue the experiment. Listed during the experiment M 、 N with ΔS the price of the option obtained when taking different values.

Table. 1 different M 、 N with ΔS Value, different price of the option

M	N	ΔS	C_0
20	10	5	3.9113
20	50	5	3.9662
40	50	2.5	4.0395
40	100	2.5	4.0461
100	50	1	4.0595
100	100	1	4.0659
100	200	1	4.0691

Can be found from the above table M Certain time, N The bigger the C_0 Closer to the true value, when N for sure, M The bigger the same, the same C_0 Closer to the true value, the more the period of the option is, the slower the stock price rises, that is, the smaller the step is taken, the closer the result is to the true value.The final picture of the price of each option is as follows:

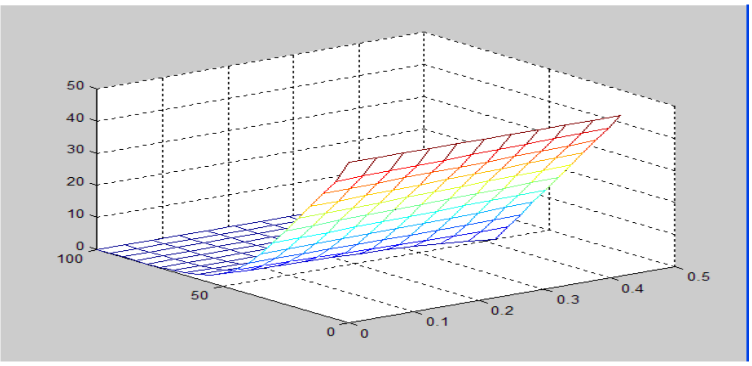

Figure. 1 Option price description chart

4. Conclusion

Since the reform and opening up 30 years ago, China's links with the international financial community have become more and more close. How to prevent and resolve financial risks has caused great concern about the face-to-face.Since 1995, China's options market has only been developed for more than a decade, but the demand for the options market is quite mature.The predecessors have conducted in-depth research on European option pricing. In 1973, Fischer Black and Myron Scholes established the call option pricing formula and won the Nobel Prize.The focus of this paper is based on the discussion and analysis of the model and numerical method of option pricing, and examples to help highlight its applicability.

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